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3.55 keV X-ray line signal from excited dark matter in radiative neutrino model

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ABSTRACT

We study an exciting dark matter scenario in a radiative neutrino model to explain the X-ray line signal at 3.55 keV recently reported by XMN-Newton X-ray observatory using data of various galaxy clusters and Andromeda galaxy. We show that the required large cross section for the up-scattering process to explain the X-ray line can be obtained via the resonance of the pseudo-scalar. Moreover, this model can be compatible with the thermal production of dark matter and the constraint from the direct detection experiment.

1. Introduction

In the light of anomalous X-ray line signal at 3.55 keV from the analysis of XMN-Newton X-ray observatory data of various galaxy clusters and Andromeda galaxy [1,2], dark matter (DM) whose mass is in the range from keV to GeV comes into one of the promising candidates. Subsequently, a number of literatures recently arose around the subject [3–22]. As for the keV scale DM, for example, a sterile neutrino can be one of the typical candidates to explain the X-ray anomaly that requires tiny mixing between the DM and the active neutrino; $\sin^2 2\theta \approx 10^{-10}$ [1]. However, these scenarios suggest that neutrino masses cannot be derived consistently with the sterile neutrino DM due to its too small mixing. Moreover, the sterile neutrino DM mass is out of the range in the direct detection searches such as LUX [23], which is currently the most powerful experiment to constrain the kind of Weakly Interacting Massive Particle.

As for the GeV scale DM, on the other hand, the exciting DM scenario which requests a pair of ground state and excited DM is known to explain the X-ray [7]. In this framework, the emission of X-ray is simply realized as follows. After the ground state DM up-annihilates into the excited DM pair, it can decay into photons (X-ray) and the ground state DM. The mass difference among them is assumed to be the energy of the X-ray, 3.55 keV. Since the framework of the exciting DM is simple, this scenario can be applicable to various models such as radiative neutrino models [24–27].

Table 1
The new particle contents and the charges for bosons where $i = 1$–3 is generation index.

Particle	L_i	e_i	N_i	η	Φ	Σ
$(SU(2)_L, U(1)_Y)$	$(\mathbf{2}, -1/2)$	$(\mathbf{1}, -1)$	$(\mathbf{1}, 0)$	$(\mathbf{2}, 1/2)$	$(\mathbf{2}, 1/2)$	$(\mathbf{1}, 0)$
Z_3	ω^2	1	ω	ω^2	ω^2	ω
Z_2	+	+	−	−	+	+

In this kind of models, small neutrino masses and existence of DM would be accommodated unlike the sterile neutrino DM scenarios above. Moreover, the DM can be testable in direct detection searches because the DM mass is GeV scale.

In this Letter, we account for the X-ray anomaly in terms of an excited DM scenario in a simple extended model with radiative neutrino masses [25], in which three right-handed neutrinos, a $SU(2)_L$ doublet scalar and a singlet scalar are added to the Standard Model (SM) and the first two lightest right-handed neutrinos are assumed to be a pair of ground state and excited state DM.

2. The model

2.1. Model setup

The particle contents and charge assignments of the model we consider are shown in Table 1. We introduce three right-handed neutrinos N_i ($i = 1$–3) where the first two lightest ones are identified to be a pair of ground state and excited state DM. We also introduce a $SU(2)_L$ doublet inert scalar η that is assumed not to have vacuum expectation value (VEV), and a gauge singlet boson Σ with non-zero VEV in addition to the SM like Higgs boson Φ. The Z_2 symmetry is imposed to assure the stability

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of DM. The Z_3 symmetry plays an important role in forbidding the term $(\Sigma+\Sigma^\dagger)\overline{N_i^c}P_R N_i$ that leads no pseudo-scalar coupling like $\Sigma_I\overline{N_i^c}\gamma_5 N_i$ where Σ_I is the imaginary part of Σ. As we will see later, the pseudo-scalar coupling is important to induce up-scattering process $N_1N_1\to N_2N_2$. The Z_3 symmetry also allows the cubic term $\Sigma^3+\text{h.c.}$ that provides the mass of the pseudo-scalar component of Σ. The relevant Lagrangian for the discussion is given as follows

$$\begin{aligned}\mathcal{L}&=\left(D^\mu\Phi\right)^\dagger(D_\mu\Phi)+\left(D^\mu\eta\right)^\dagger(D_\mu\eta)\\&\quad+\left(y_\ell\bar{L}\Phi e+y_\eta\bar{L}\eta^\dagger N+\frac{y_N}{2}\Sigma\bar{N}^c N+\text{h.c.}\right),\\\mathcal{V}&=m_1^2\Phi^\dagger\Phi+m_2^2\eta^\dagger\eta+m_3^2\Sigma^\dagger\Sigma+\left(\mu\Sigma^3+\text{h.c.}\right)\\&\quad+\lambda_1\left(\Phi^\dagger\Phi\right)^2+\lambda_2\left(\eta^\dagger\eta\right)^2+\lambda_3\left(\Phi^\dagger\Phi\right)\left(\eta^\dagger\eta\right)\\&\quad+\lambda_4\left(\Phi^\dagger\eta\right)\left(\eta^\dagger\Phi\right)+\left[\lambda_5\left(\Phi^\dagger\eta\right)^2+\text{h.c.}\right]\\&\quad+\lambda_6\left(\Sigma^\dagger\Sigma\right)^2+\lambda_7\left(\Sigma^\dagger\Sigma\right)\left(\Phi^\dagger\Phi\right)+\lambda_8\left(\Sigma^\dagger\Sigma\right)\left(\eta^\dagger\eta\right),\end{aligned}\tag{2.1}$$

where the generation indices are omitted, and the Yukawa coupling y_N can be regarded as diagonal in general. After the electroweak symmetry breaking, the scalar fields can be parametrized as

$$\begin{aligned}&\Phi=\begin{pmatrix}G^+\\\frac{v+\phi^0+iG^0}{\sqrt{2}}\end{pmatrix},\qquad\eta=\begin{pmatrix}\eta^+\\\frac{1}{\sqrt{2}}(\eta_R+i\eta_I)\end{pmatrix},\\&\Sigma=\frac{v'+\sigma+i\rho}{\sqrt{2}},\end{aligned}\tag{2.2}$$

where $v\approx 246$ GeV, and G^+ and G^0 are absorbed in W^+ boson and Z boson due to the Higgs mechanism. The resulting CP-even mass matrix with nonzero VEV is given by

$$m^2(\phi^0,\sigma)=\begin{pmatrix}2\lambda_1 v^2 & \lambda_7 v v'\\ \lambda_7 v v' & \frac{(3\sqrt{2}\mu+4\lambda_6 v')v'}{2}\end{pmatrix},\tag{2.3}$$

where the tadpole conditions $\partial\mathcal{V}/\partial\phi^0|_{\text{VEV}}=0$ and $\partial\mathcal{V}/\partial\sigma|_{\text{VEV}}=0$ are inserted. This mass matrix is diagonalized by the rotation matrix, and ϕ^0 and σ are rewritten by the mass eigenstates h and H as

$$\begin{aligned}\phi^0&=h\cos\alpha+H\sin\alpha,\\\sigma&=-h\sin\alpha+H\cos\alpha.\end{aligned}\tag{2.4}$$

The mass eigenstate h corresponds to the SM-like Higgs and H is an extra Higgs respectively. The mixing angle $\sin\alpha$ is expressed as the function in terms of the other parameters as

$$\sin 2\alpha=\frac{\lambda_7 v v'}{m_h^2-m_H^2}.\tag{2.5}$$

The pseudo-scalar ρ does not mix after the symmetry breaking and the mass is just given by $m_\rho^2=9\mu^2/\sqrt{2}$. The masses of the other Z_2 odd scalars η^+, η_R and η_I are also determined adequately to be

$$m_\eta^2=m_2^2+\frac{1}{2}\lambda_3 v^2+\frac{1}{2}\lambda_8 v'^2,\tag{2.6}$$

$$m_R^2=m_2^2+\frac{1}{2}\lambda_8 v'^2+\frac{1}{2}(\lambda_3+\lambda_4+2\lambda_5)v^2,\tag{2.7}$$

$$m_I^2=m_2^2+\frac{1}{2}\lambda_8 v'^2+\frac{1}{2}(\lambda_3+\lambda_4-2\lambda_5)v^2.\tag{2.8}$$

The mass splitting between m_R and m_I is given by $m_R^2-m_I^2=2\lambda_5 v^2$. The lower bounds of the inert scalar masses are obtained as $m_\eta\gtrsim 70$ GeV and $m_R, m_I\geq 45$ GeV by the LEP experiment [29–31] and the invisible decay of Z boson [31]. In addition, the mass difference between the charged and neutral inert scalars is constrained as roughly less than $\mathcal{O}(100)$ GeV by the T parameter [28].

2.2. Neutrino sector

The right-handed neutrinos obtain the masses after the symmetry breaking due to VEV of Σ,

$$M=\frac{v'}{\sqrt{2}}\begin{pmatrix}y_{N1}&0&0\\0&y_{N2}&0\\0&0&y_{N3}\end{pmatrix}\equiv\begin{pmatrix}M_1&0&0\\0&M_2&0\\0&0&M_3\end{pmatrix}.\tag{2.9}$$

Using the right-handed neutrino masses, the active neutrino masses can be obtained at one-loop level as [25]

$$\begin{aligned}(m_\nu)_{ab}&=\sum_i\frac{(y_\eta)_{ai}(y_\eta)_{bi}M_i}{2(4\pi)^2}\\&\quad\times\left[\frac{m_R^2}{m_R^2-M_i^2}\ln\frac{m_R^2}{M_i^2}-\frac{m_I^2}{m_I^2-M_i^2}\ln\frac{m_I^2}{M_i^2}\right].\end{aligned}\tag{2.10}$$

In particular, when the mass splitting between η_R and η_I is small ($\lambda_5\ll 1$) and N_i are much lighter than η ($M_i\ll m_R\approx m_I$), the formula can be simplified as follows

$$(m_\nu)_{ab}\approx\frac{\lambda_5 v^2}{(4\pi)^2(m_R^2+m_I^2)}\sum_i(y_\eta)_{ai}(y_\eta)_{bi}M_i.\tag{2.11}$$

We will consider the mass hierarchy for the analysis in the next section. The following parameter set is taken for example to be consistent with the sum of the light neutrino masses 0.933 eV [32]

$$\begin{aligned}&M\sim\mathcal{O}(10)\text{ GeV},\qquad y_\eta\approx 0.1,\qquad\lambda_5\approx 10^{-5},\\&m_R\approx m_I\sim\mathcal{O}(1)\text{ TeV}.\end{aligned}\tag{2.12}$$

Note that the Yukawa coupling y_η cannot be too small since the lifetime of the decay channel $N_2\to N_1\gamma$ becomes too long to explain the X-ray anomaly.

Lepton Flavor Violating processes such as $\mu\to e\gamma$ or $\mu\to 3e$ should be taken into account [33]. One may think that the above parametrization has been already excluded by the strong constraint of $\mu\to e\gamma$ whose branching ratio should be $\text{Br}(\mu\to e\gamma)\leq 5.7\times 10^{-13}$. However, it can be evaded by considering a specific flavor structure of the Yukawa coupling y_η as Refs. [34–36].

3. Dark matter

We identify that N_1 and N_2 are a pair of ground and excited state DM for explaining the X-ray anomaly. Thus their masses are related as $M_1\approx M_2<M_3$, and $M_2-M_1\equiv\Delta M=3.55$ keV. Such the situation has been considered for a different motivation in Refs. [34–36]. The small mass splitting between N_1 and N_2 would be theoretically derived by introducing an extra $U(1)$ symmetry. For example, we can construct the model that the interactions ΣN_1N_1 and ΣN_2N_2 are forbidden but ΣN_1N_2 is allowed, and the small $U(1)$ breaking terms such as N_1N_1 and N_2N_2 come from higher-dimensional operators. Then after diagonalizing the mass matrix composed by N_1 and N_2, almost degenerated two mass eigenstates are obtained.

A small momentum of DM is required to lead the up-scattering event $N_1N_1\to N_2N_2$. To induce the up-scattering process, the required minimum relative velocity of a pair of DM is estimated as $v_{\min}\approx 2\sqrt{2\Delta M/M_1}$ from the kinematics. It suggests that the mass of DM should be $\mathcal{O}(10)$ GeV since the averaged DM velocity in the

present universe is estimated as $v_{\rm rel} \sim 10^{-3}$. The local photon flux is approximately given by [7]

$$F \approx 2.6 \times 10^{-5} \left[\frac{\langle \sigma v_{\rm rel}(N_1 N_1 \to N_2 N_2) \rangle}{10^{-19}\ {\rm cm}^3/{\rm s}} \right] \times \left[\frac{10\ {\rm GeV}}{M_1} \right]^2 {\rm photon/s}, \tag{3.1}$$

where NFW profile is assumed [38]. The above formula suggests that we need a quite large cross section to explain the X-ray line. However, our model can obtain such a large cross section via the ρ resonance.

The up-scattering process $N_1 N_1 \to N_2 N_2$ is derived by the massive pseudo-scalar ρ. Since the required cross section for the process is very large as $\sigma v_{\rm rel} \sim 10^{-19}$ cm^3/s [7], an enhancement mechanism is required.[1] In our case, we have the resonance in the ρ mediated s-channel which leads s-wave for the cross section. In this sense, the interaction between the pseudo-scalar ρ and two DM are crucially important. The up-scattering cross section is given by

$$\sigma v_{\rm rel}(N_1 N_1 \to N_2 N_2) \approx \frac{|y_{N1} y_{N2}|^2 s}{16\pi} \sqrt{1 - \frac{4M_2^2}{s}} \frac{1}{(s - m_\rho^2)^2 + \Gamma_\rho^2 m_\rho^2}, \tag{3.2}$$

with

$$\Gamma_\rho = \frac{|y_{N1}|^2}{16\pi} m_\rho \sqrt{1 - \frac{4M_1^2}{m_\rho^2}}, \tag{3.3}$$

where $s \approx 4M_1^2(1 + v_{\rm rel}^2/4)$. We define the mass difference between $2m_\rho$ and M_1 as $\Delta \equiv 1 - m_\rho^2/4M_1^2$, and focus on the physical pole $\Delta > 0$. The cross section should be velocity averaged since the velocity of DM has a distribution in the DM halo. Assuming the Maxwell–Boltzmann distribution, the velocity averaged cross section is given by [37]

$$\langle \sigma v_{\rm rel} \rangle = \frac{1}{2\sqrt{\pi} v_0^3} \int_0^\infty v_{\rm rel}^2 (\sigma v_{\rm rel}) e^{-v_{\rm rel}^2/4v_0^2} dv_{\rm rel}, \tag{3.4}$$

where $v_0 = 10^{-3}$ is the dispersion of the DM velocity. The contours of the velocity averaged cross section are shown in Fig. 1. The figure suggests, for example, that the required large cross section $\langle \sigma v_{\rm rel} \rangle \sim 10^{-19}$ cm^3/s can be realized when Δ is $10^{-4} \lesssim \Delta \lesssim 10^{-3}$ in the DM mass range 1 GeV $\lesssim M_1 \lesssim$ 100 GeV. Note that this cross section into $N_2 N_2$ does not contribute to estimate the relic density of DM.

After the up-scattering, N_2 immediately decays into the ground state DM (N_1) and photon through η^+ at one-loop level which is the dominant decay process of the excited DM. The decay width of the process $N_2 \to N_1 \gamma$ is calculated as

$$\Gamma(N_2 \to N_1 \gamma) = \frac{\mu_{12}^2}{\pi} \Delta M^3, \tag{3.5}$$

where μ_{12} is the transition magnetic moment between N_1 and N_2 which is calculated as [36]

$$\mu_{12} \approx \sum_a \frac{{\rm Im}[(y_\eta^*)_{a1}(y_\eta)_{a2}] e M_1}{2(4\pi)^2 m_\eta^2}. \tag{3.6}$$

[1] Note here that the pair of η_I and η_R cannot be used to explain the X-ray line because the decay process $\eta_R \to \eta_I \gamma$ is forbidden by spin statistics.

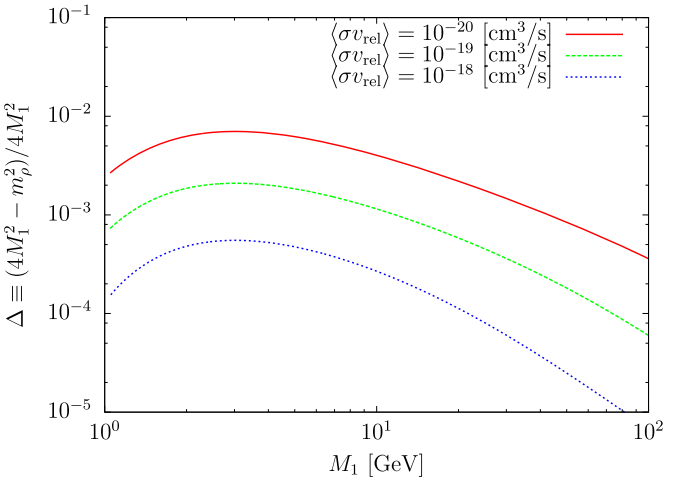

Fig. 1. Contours of velocity averaged cross section in M_1–Δ plane. We find that rather mild fine tuning provides the required cross section over the range of DM mass depicted in figure. Here we fix as $y_{N1} = y_{N2} \approx 1$.

where e is the electromagnetic coupling constant. The lifetime of the excited DM N_2 should be much less than the cosmological timescale $\tau \sim 10^{17}$ s so as to decay immediately after the N_2 production. From the requirement, the order of the Yukawa coupling y_η is estimated as $y_\eta \gtrsim 0.01$. This does not conflict with the parameter set of Eq. (2.12). As one can see from Eq. (3.6), a complex phase of the Yukawa coupling y_η is necessary to induce the decay $N_2 \to N_1 \gamma$.

Next we consider the thermal relic density of DM. The cross section contributing to the relic density is dominantly given via h and H s-channel. Although there the other contributions through t and u-channel via η exchange [39,40], these contributions are negligible due to the heavy mass of the intermediate state η. Hence we focus on only the s-channel contribution. The cross section for the channel $N_1 N_1 \to f\bar{f}$ mediated by h and H is given by

$$\sigma v_{\rm rel} \simeq \frac{3 y_{N1}^2 y_b^2 M_1^2 \sin^2 2\alpha}{256\pi} \times \left| \frac{1}{s - m_h^2 + i m_h \Gamma_h} - \frac{1}{s - m_H^2 + i m_H \Gamma_H} \right|^2 \times \left(1 - \frac{m_b^2}{M_1^2} \right)^{3/2} v_{\rm rel}^2, \tag{3.7}$$

where only bottom pair is taken into account in fermion pair $f\bar{f}$ due to the kinematics and strength of Yukawa coupling. As one can see from the equation, we have only p-wave contribution. The co-annihilation with N_2 should be taken into account since the masses among them are degenerated. However, the order of the effective cross section including the co-annihilation process is same with Eq. (3.7) as long as $y_{N1} \approx y_{N2}$ is assumed. The SM-like Higgs decay width are fixed to be $\Gamma_h = 4.1 \times 10^{-3}$ GeV [41]. When we consider the mass hierarchy $m_H \gtrsim 2M_1$, the dominant decay width of the second Higgs Γ_H is expressed as

$$\Gamma_H = \frac{y_{N1}^2 \cos^2\alpha}{16\pi} m_H \left(1 - \frac{4M_1^2}{m_H^2} \right)^{3/2}. \tag{3.8}$$

To obtain the correct relic density $\Omega h^2 \approx 0.12$, the required cross section is $\sigma v_{\rm rel} \approx 3 \times 10^{-26}$ cm^3/s. In the left panel of Fig. 2, the contours of the required cross section for the thermal relic density are plotted in the plane of DM mass and the mass degeneracy between DM and H. As one can see, stronger degeneracy between

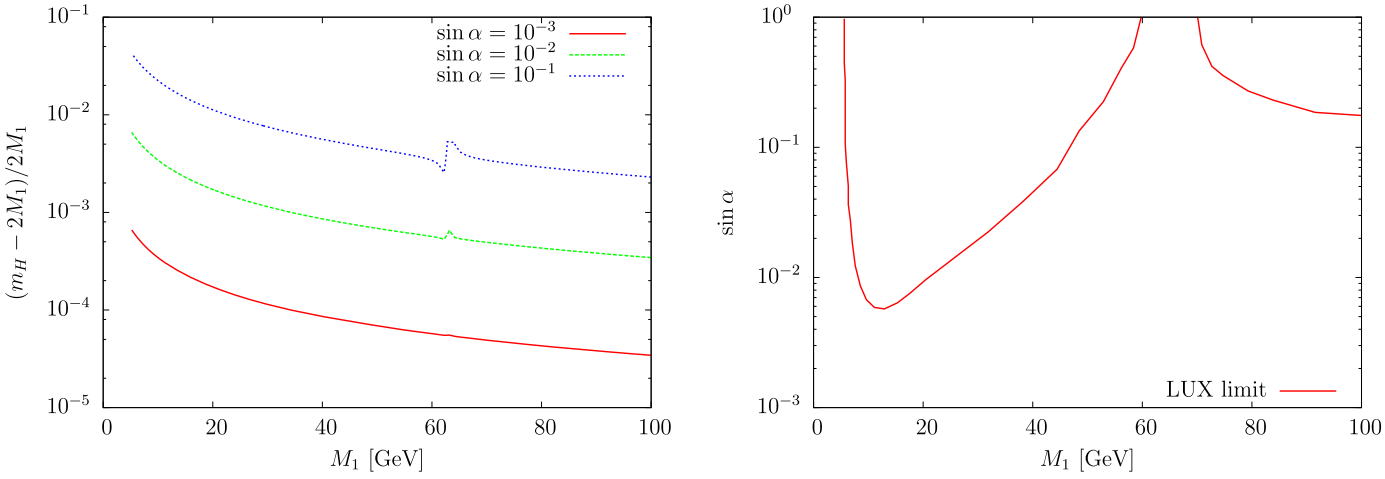

Fig. 2. Contours of required cross section for thermal relic density of DM (left panel) and constraint on mixing angle $\sin\alpha$ from direct detection (right panel).

DM and H is necessary for smaller mixing angle $\sin\alpha$ in order to induce the appropriate cross section for the thermal relic density. The peak at $M_1 \approx 63$ GeV is due to the SM Higgs resonance $2M_1 \approx m_h$.

The direct detection constraint also should be considered since the scale of our DM is GeV. The elastic cross section with proton is induced by the t-channel Higgs mediation and it is calculated as

$$\sigma_p = \frac{C y_{N1}^2 \sin^2 2\alpha}{8\pi v^2} \frac{m_p^4 M_1^2}{(m_p + M_1)^2} \left(\frac{1}{m_h^2} - \frac{1}{m_H^2} \right)^2, \tag{3.9}$$

where $C \approx 0.079$. At present, the LUX experiment gives the strongest constraint on the elastic cross section. The constraint of the LUX experiment can be translated to the constraint on the mixing angle $\sin\alpha$ in our case as shown in the right panel of Fig. 2. In the figure, the mass of the second Higgs m_H is fixed to $m_H = 2M_1$ from the requirement of the thermal relic density. One can see that the mixing angle $\sin\alpha$ should be $\sin\alpha \lesssim 0.005$ in order to evade the direct detection constraint in whole DM mass range.

4. Summary and conclusion

We have studied an exciting DM scenario in a radiative neutrino model to explain the X-ray line signal at 3.55 keV recently reported by XMN-Newton X-ray observatory using data of various galaxy clusters and Andromeda galaxy. We have shown that neutrino masses can be radiatively generated by our DM with the mass of $\mathcal{O}(10)$ GeV, which is requested by the exciting DM scenario. Also we have shown that the required large cross section to explain the X-ray line can be obtained by the resonance of the massive pseudo-scalar ρ that provides s-wave contribution for only the up-scattering process $N_1 N_1 \to N_2 N_2$. The model can be consistent with the observed relic density as well as the direct detection constraint. To induce 3.55 keV X-ray line without any inconsistencies, we have found that the mass degeneracies $m_H \approx m_\rho \approx 2M_1$ are required in the model.

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